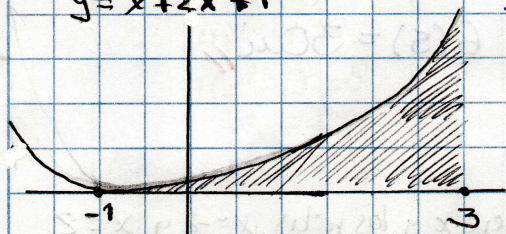


# I) DETERMINAR EL ÁREA DE LA REGIÓN PARA CADA CASO, EMPLEANDO DEFINICIÓN DE INTEGRAL DEFINIDA:

1) La región plana acotada por  $y = x^2 + 2x + 1$ , el eje  $x$  y las rectas  $x = -1$  y  $x = 3$

$$y = x^2 + 2x + 1$$



$$i) \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x = \int_{-1}^3 x^2 + 2x + 1 dx$$

$$ii) \Delta x = \frac{b-a}{n} = \frac{4}{n}$$

$$iii) x_i = a + i \Delta x = \frac{4i}{n} - 1$$

$$iv) \lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(\frac{4i}{n} - 1\right) \frac{4}{n} = \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[ \left(\frac{4i}{n} - 1\right)^2 + 2\left(\frac{4i}{n} - 1\right) + 1 \right] \frac{4}{n}$$

$$\cdot \left(\frac{4i}{n} - 1\right)^2 = \frac{16i^2}{n^2} - \frac{8i}{n} + 1$$

$$\cdot 2\left(\frac{4i}{n} - 1\right) = \frac{8i}{n} - 2$$

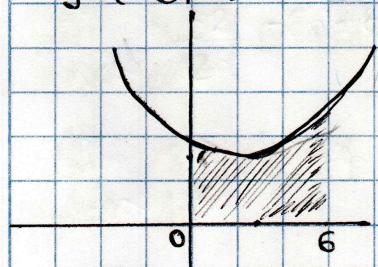
$$\lim_{n \rightarrow \infty} \frac{4}{n} \sum_{i=1}^n \left( \frac{16i^2}{n^2} - \frac{8i}{n} + 1 - \frac{8i}{n} + 2 + 1 \right) = \lim_{n \rightarrow \infty} \frac{4}{n} \sum_{i=1}^n \frac{16i^2}{n^2}$$

$$\lim_{n \rightarrow \infty} \frac{64}{n^3} \sum_{i=1}^n i^2 = \lim_{n \rightarrow \infty} \frac{64}{n^3} \frac{(n(n+1)(2n+1))}{6} = \frac{64}{6} \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^2 + n}{n^3}$$

$$\frac{64}{6} \lim_{n \rightarrow \infty} \frac{n^3 \left(2 + \frac{3}{n} + \frac{1}{n}\right)}{n^3} = \frac{64}{6} (2) = \frac{64}{3}$$

2) La región plana acotada por  $y = (x-3)^2 + 2$ , el eje  $x$  y las rectas  $x = 0$  y  $x = 6$

$$y = (x-3)^2 + 2$$



$$i) \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x = \int_0^6 (x-3)^2 + 2 dx$$

$$ii) \Delta x = \frac{b-a}{n} = \frac{6}{n} \quad x_i = a + i \Delta x = \frac{6i}{n}$$

$$iii) \lim_{n \rightarrow \infty} \sum_{i=1}^n f\left(\frac{6i}{n}\right) \frac{6}{n} = \lim_{n \rightarrow \infty} \frac{6}{n} \sum_{i=1}^n \left(\frac{6i}{n} - 3\right)^2 + 2$$

$$\cdot \sum_{i=1}^n \frac{1}{n^2} (6i-3n)^2 + 2 = \sum_{i=1}^n \frac{36i^2}{n^2} - \frac{36in}{n^2} + \frac{9n^2}{n^2} + 2 = \sum_{i=1}^n \frac{36i^2}{n^2} - \frac{36i}{n} + 11$$

$$\cdot \frac{36}{n^2} \sum_{i=1}^n i^2 - \frac{36}{n} \sum_{i=1}^n i + 11 \sum_{i=1}^n 1 = \frac{36}{n^2} \frac{(n(n+1)(2n+1))}{6} - \frac{36(n(n+1))}{n} + 11n$$

$$\cdot \frac{6(n+1)(2n+1) - 36(n+1) + 11n^2}{n}$$